201. Two submarines leave port simultaneously. One travels on bearing 090° at 15 mph, the other on bearing 340° at 18 mph. Determine the distance between the submarines after 20 minutes.

202. A line passes through points $(b, 2b)$ and $(-b, 4b)$, where $b \neq 0$. Find the acute angle between this line and the $x$ axis.

203. The number of tails $X$ when four coins are tossed is described as following a binomial distribution:

$$X \sim B(4, 1/2).$$

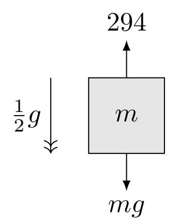
Prove, without quoting any results pertaining to the binomial distribution, that $\mathbb{P}(X = 2) = 3/8$.

204. "The $x$ axis is tangent to the curve $y = x^2 - 6x + 9$." True or false?

205. State, with a reason, whether the following gives a well-defined function:

$$\text{f} : \begin{cases} \mathbb{R} \mapsto \mathbb{R} \\ x \mapsto \dfrac{1}{x}. \end{cases}$$

206. An object of mass $m$ kg has forces acting on it as shown in the diagram. Forces are in Newtons, and acceleration in $\text{ms}^{-2}$.




Determine the value of $m$.

207. Simplify $\dfrac{x^4 - 1}{x^2 - 1}$.

208. The sum of the first $n$ integers is given by

$$\text{S}(n) = \tfrac{1}{2}n(n + 1).$$

Verify this result by showing that

$$\text{S}(n - 1) + n = \text{S}(n).$$

209. Events $A$ and $B$, which are mutually exclusive, have probabilities $p_1$ and $p_2$. Of the following three equations, one cannot possibly be true and one produces a named relationship between events $A$ and $B$. Identify and explain these two equations.

① $p_1 + p_2 = \frac{4}{5}$,
② $p_1 + p_2 = 1$,
③ $p_1 + p_2 = \frac{5}{4}$.

210. Find the exterior angle of a regular decagon, giving your answer exactly in radians.

211. A sample $\{x_i\}$ is given as follows:

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 |
| $f$ | 6 | 14 | 9 | 2 |

Use the statistical functions on your calculator to find the mean $\bar{x}$ and standard deviation $s$.

212. A curve has gradient formula $\dfrac{dy}{dx} = 3x^2 + 1$.

(a) Show that $y = ax^3 + bx^2 + cx + d$, where $a, b, c$ are constants to be determined.
(b) The curve passes through $(-1, 5)$. Find $d$.

213. An arithmetic sequence has $n$th term $u_n$. Show that the sequence given by the following ordinal formula is also arithmetic:

$$w_n = u_{n-1} + u_n.$$

214. The straight line segment $x = 2t$, $y = 1 - t$, for $t \in [0, 5]$ is reflected in the line $y = x$. Write down, without doing any calculations, the equation of the new line, in the same form.

215. Show that the following claim is not true: "It is impossible for an object to remain in equilibrium under the action of five forces, each of which has magnitude $F$."

216. Find the equation of the line through $(a, a+1)$ and $(a + 2, a + 3)$.

217. A student suggests that the following is an identity for some suitable choice of constants $A, B$:

$$x(x - 1)(x + 1) \equiv Ax^2(x - 1) + B(x^2 - 1).$$

By multiplying out and comparing the coefficients of powers of $x$, or otherwise, prove that the student is mistaken.

218. A functional instruction is given by

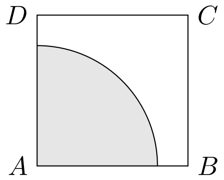
$$\text{f}(x) = \frac{2}{x - 4}.$$

(a) The largest real domain over which f may be defined is $\mathbb{R} \setminus \{a\}$. Write down the value of $a$.
(b) By setting $\text{f}(x)$ equal to $y$ and rearranging to make $x$ the subject, determine the functional instruction $\text{f}^{-1}(x)$.

219. Solve the following simultaneous equations:

$$x^2 + y^2 = 1,$$
$$y = \tfrac{1}{2}x + 1.$$

220. In a square $ABCD$ of side length 1 cm, 50% of the area is within $a$ cm of vertex $A$.



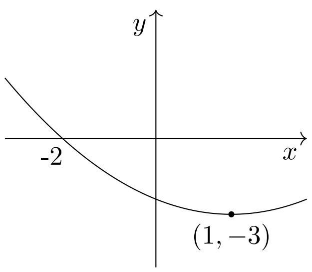

Determine the exact value of $a$.

221. The numbers 1 to 5 are randomly assigned to the vertices of a pentagon. Find the probability that 1 and 2 are adjacent.

222. Three forces, with magnitudes $10, 20, 40$ N, act on a particle of mass 5 kg. It accelerates at 2 ms$^{-2}$. Show that all three forces must have the same line of action.

223. Show that the points of intersection of the graphs $y = 2x + 5$ and $xy = 3$ are a distance $7\sqrt{5}/2$ apart.

224. Find the equation of the parabola shown below, on which a root and the vertex have been marked.




Give your answer in the form $3y = ax^2 + bx + c$.

225. A die is rolled repeatedly until a six is attained. Show that the probability of requiring a total of $r$ rolls to attain a six is

$$\mathbb{P}(r) = \frac{5^{r-1}}{6^r}.$$

226. A curve is defined by $y = 2^x - 4^x$.

(a) Show that $y = 0$ is an asymptote.

(b) Find the axis intercepts of the curve.

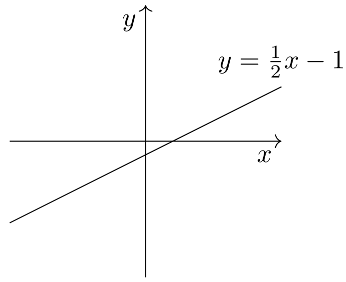
(c) Hence, sketch the curve.

227. By constructing a six-by-six grid representing the possibility space, find the probability that, when two dice are rolled, both show prime numbers.

228. The terms of an AP have mean 10 and sum 420. Find the number of terms in the progression.

229. Simplify $\dfrac{x+a}{b-x} \times \dfrac{a-x}{x+b} \times \dfrac{x^2-b^2}{x^2-a^2}$.

230. The graph below is of $y = f'(x)$, for some function f defined over the real numbers.




Find all possible functions f.

231. Disprove the following statements by providing a counterexample to each:

(a) "No kite is also a parallelogram."

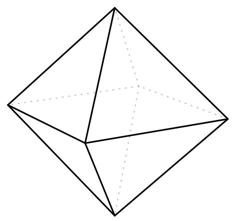
(b) "A prime number cannot be a multiple of 11."

232. The locus $L$ is defined as the boundary of the set of points which are closer to $(4, 0)$ than they are to $(0, 0)$. Find the equation of $L$.

233. Solve the equation $x^3 = 4x$.

234. You are given that the lines $2x + 3y = 5$ and $kx - 4y = 1$ are perpendicular to one another. Determine the value of $k$.

235. A regular octahedron has eight faces, each of which is an equilateral triangle.



Prove that the surface area of a regular octahedron is $2\sqrt{3}a^2$, where $a$ is edge length.

236. A sample of size 24 has mean $\bar{x} = 4$ and standard deviation $s = 1.2$. A further datum is then added to the sample, which causes an alteration to the standard deviation, but not to the mean. Explain whether it is possible to determine the value of the new datum from this information.

237. Evaluate $\displaystyle\sum_{k=1}^{4} \cos(90k - 15)^\circ$.

238. An expression is given as $\left(x^2 - a^2\right)\left(x^2 + b^2\right)$, where $a$ and $b$ are non-zero constants and $x$ is a variable. Write down the roots of this expression.

239. An extreme runner is running across a glacier at 6 $\text{ms}^{-1}$ when he spots a small ravine ahead. The ravine is 3 metres wide. He decides to jump it, without stopping. Assuming his horizontal speed remains constant, determine the vertical speed with which he must leap to make it safely across.

240. In an attempted solution, a student writes

$$\begin{aligned} & 2x^2 - 3x > x^2 + x \\ \Longrightarrow\ & x^2 - 4x > 0 \\ \Longrightarrow\ & x - 4 > 0 \\ \Longrightarrow\ & x > 4. \end{aligned}$$

Explain the error, giving a counterexample to the relevant implication, and correct the solution.

241. If $f'(x) = \dfrac{1+x}{\sqrt{x}}$, find and simplify $f''(x)$.

242. Two forces, with magnitudes 8 and 15 Newtons, act in perpendicular directions on an object of mass 34 kg. Find the acceleration of the object.

243. Prove that the product $pq$ of two rational numbers $p$ and $q$ is rational.

244. State, with a reason, whether $y = 2x + k$ intersects the following lines:

(a) $y = 1 - 2x + k$,

(b) $y = 1 + 2x + k$.

245. Solve $(x^2 + 1)^2 - (x^2 - 1)^2 = 0$

246. The definite integral below gives the displacement, over a particular time period, for an object moving with constant speed:

$$s = \int_2^5 8\, dt.$$

Write down the speed and duration of the motion, and calculate the displacement.

247. Prove that, for all positive real numbers $x, y$,

$$x + y > 5 \implies 2x + 3y > 10.$$

248. The point $(\sqrt{20}, 0)$ is rotated anticlockwise around the origin by an angle $\theta = \arctan \frac{1}{2}$.

(a) Explain why the new coordinates must be of the form $(2k, k)$, for some positive $k$.

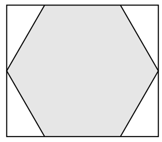
(b) Find the new coordinates.

249. £1500 is invested at an annual interest rate of 4.3%, compounded every quarter. Show that the total interest received after 6 years is £438.84.

250. A function g is such that $g'(x) = \frac{1}{3}$ for all real numbers $x$. Find $g(6) - g(0)$.

251. Prove that the product of 7 consecutive integers is divisible by 7.

252. A rectangle is drawn around a regular hexagon.



Show that the ratio of the lengths of the sides of the rectangle is $2 : \sqrt{3}$.

253. If $\log_3 y = x$, write $9^x$ in terms of $y$.

254. Find the probability that, if two letters are chosen at random from the word DEUCE, neither is E.

255. Prove that, in a plane, precisely six unit circles may be placed around a seventh, such that each is tangent to the central circle and its neighbours.

256. A parabola has equation $y = 2x^2 - 3x + 6$.

(a) Find the gradient formula $\dfrac{dy}{dx}$.

(b) Evaluate the gradient formula at $x = 2$.

(c) Find the $y$ coordinate at $x = 2$.

(d) Hence, find the equation of the tangent to the curve at $x = 2$.

257. Using position vectors, prove that, for any three points $A, B, C$, the line joining the midpoint of $AB$ to the midpoint of $AC$ is parallel to $BC$.

258. The definition of ${}^n\mathrm{C}_r$ is

$${}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}.$$

From this definition, show that

(a) ${}^n\mathrm{C}_1 \equiv n$,

(b) ${}^n\mathrm{C}_{n-2} \equiv \frac{1}{2}n(n-1)$, for $n \geq 2$.

259. State, giving a reason, which of the implications $\Longrightarrow$, $\Longleftarrow$, $\Longleftrightarrow$ links the following statements concerning a real number $x$:

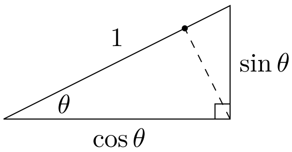
① $(x-a)(x-b)(x-c) = 0$,

② $x \in \{a, b, c\}$.

260. Express $x^2 + 2x + 5$ in terms of $(x - 1)$.

261. Write down the equation of the locus of all points which are equidistant from $(4, 0)$ and $(0, 4)$.

262. The triangle below defines the functions sine and cosine over the domain $(0°, 90°)$. A perpendicular has been drawn to the hypotenuse.




(a) Show that the dashed perpendicular has length $\sin\theta\cos\theta$.

(b) The marked point splits the hypotenuse into two sections. Find, in terms of $\theta$, the lengths of these sections, and give the trigonometric identity which is thus verified.

263. In the following equation, the constants $p, q, r$ and $s$ are all distinct and satisfy $p, q, r, s > 0$:

$$\frac{(x^2 - p^2)(x^2 + q^2)}{(x^2 - r^2)(x^2 + s^2)} = 0.$$

Write down the roots of the equation.

264. Explain the error in the following argument: "In zero gravity, there can be no reaction forces on an object, because reaction forces exist in reaction to gravity."

265. Find the value of the constant $k$, if the following curve has an asymptote at $x = 4$:

$$y = \frac{1}{x^2 + x + k}.$$

266. The small-angle approximation for $\cos\theta$, for $\theta$ in radians, is $\cos\theta \approx 1 - \frac{1}{2}\theta^2$. Find the percentage error in this approximation at

(a) $\theta = \frac{\pi}{24}$,

(b) $\theta = \frac{\pi}{6}$.

267. A sequence, with first term $u_0 = 1$, is defined by the iteration $u_{n+1} = 2u_n + n$. Find $u_3$.

268. Explain why the solution of the following equation is the same whatever the value of the constant $a$:

$$\frac{x^2 + 4x - 2}{x^2 + 1 + a^2} = 0.$$

269. The resultant force on a particular object of mass $m$ is given, in terms of time $t \in [0, \infty)$, by

$$F = 2m\big(t - t^2\big) \text{ N}.$$

Show that the greatest value of the acceleration is $a_{\max} = 0.5 \text{ ms}^{-2}$.

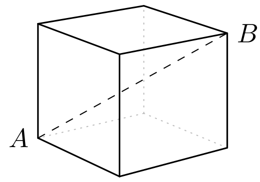
270. Sketch the curve $x = y^2 + 1$.

271. Write down the area scale factor when the curve $y = f(x)$ is transformed to the curve $y = f(3x)$.

272. Find the sum of the interior angles of a heptagon, giving your answer in radians.

273. Show that $\dfrac{\sqrt{2} - \sqrt{8}}{\sqrt{8} - \sqrt{32}} = \frac{1}{2}$.

274. The diagram shows a cube of unit side length.




Determine the distance, through the centre of the cube, between the vertices marked $A$ and $B$.

275. Write down the range of the squaring function $x \mapsto x^2$, when it is defined over the given domains:

(a) $[0, 1]$,

(b) $[-1, 1]$,

(c) $\mathbb{R}$.

276. Find the equation of the perpendicular bisector of the points $(0, 4)$ and $(12, 0)$.

277. An arithmetic progression (AP) is defined with first term $u_1 = a$ and last term $u_n = l$. Prove that the common difference $d$ is given by

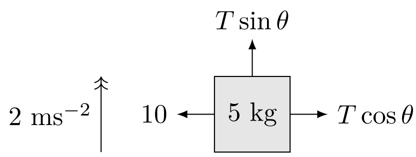
$$d = \frac{l - a}{n - 1}.$$

278. Solve for $x$ in $\dfrac{(x-a)(x+b)}{(x-a)(x+b-1)} = 0$.

279. A function has $g''(x) = 2$. By integrating twice, show that $y = g(x)$ is a monic parabola.

280. A fair die is rolled twice. By drawing a diagram of the possibility space, find the probability that the second score is higher than the first score.

281. Three forces, with magnitudes given in Newtons, cause a 5 kg mass to accelerate as depicted below:




Solve to find $T$ and $\theta$.

282. Determine whether the point $(4, 5)$ lies inside, on, or outside the ellipse $2x^2 + 3y^2 = 100$.

283. An irregular pentagon has sides whose lengths are in AP. Its perimeter is 35. Explain carefully why the length $l$ of its longest side must satisfy:

   (a) $l > 7$,

   (b) $l < 14$.

284. The exam marks of class of twenty pupils have been summarised with $\bar{x} = 69.3\%$. A new pupil then joins the class, whose mark is 86%. Find the new mean for the class.

285. For $p \neq 0$, determine the vertical asymptotes of

$$y = \frac{x^2 + p^2}{x^3 - px}.$$

286. True or false?

   (a) $x^3 = y^3 \iff x = \pm y$,

   (b) $x^4 = y^4 \iff x = \pm y$,

   (c) $x^5 = y^5 \iff x = \pm y$.

287. Give the exact area of the region of the $(x, y)$ plane defined by the inequality $(x-1)^2 + (y-2)^2 \leq 5$.

288. Two graphs have the following equations:

$$y = x^3 + x^2,$$
$$y \equiv \left(\sqrt{x} + 1\right)\left(\sqrt{x} - 1\right).$$

By differentiating, find all values of $x$ for which the gradient of the first graph is equal to the gradient of the second graph.

289. Give the range of $f(x) = (\sin x + 1)^2 + 1$.

290. A quadrilateral $ABCD$ has vertices, in order, at coordinates $(0, 0)$, $(-1, 1)$, $(2, 4)$ and $(3, 1)$.

   (a) Find the perpendicular bisectors of

      i. $AC$,

      ii. $BD$.

   (b) Find the intersection $X$ of these bisectors.

   (c) Show that $|AX| = |BX| = |CX| = |DX|$.

   (d) Hence, show that $ABCD$ is cyclic.

291. Disprove the following statement:

$$\text{If } f'(x) \equiv g'(x), \text{ then } f(x) \equiv g(x).$$

292. Show that the graphs $2x + 3y = 1$, $x - 5y = -6$ and $x^2 + y^2 = 2$ are concurrent.

293. Prove that, if two triangles have exactly $n$ points of intersection, where $n \in \mathbb{N}$, then $n \leq 6$.

294. Two of the constant acceleration formulae are true by definition.

   - **Velocity** is rate of change of displacement:

$$\tfrac{1}{2}(u + v) = \frac{s}{t},$$

   - **Acceleration** is rate of change of velocity:

$$a = \frac{v - u}{t}.$$

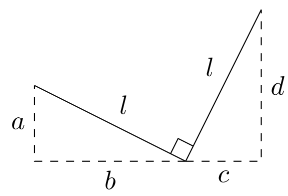
From these two, prove the formula $v^2 = u^2 + 2as$.

295. A cubic equation is given as

$$x^3 + kx^2 + 9x = 0.$$

Using the discriminant $\Delta = b^2 - 4ac$, determine all possible values of $k$ for which the equation $x^3 + kx^2 + 9x = 0$ has exactly two roots.

296. The diagram below shows gradient triangles drawn on two perpendicular lines of length $l$. Use it to prove that such gradients satisfy $m_1 m_2 = -1$.




297. Find $\int f(t)\,dt$, for

   (a) f defined by $f(t) = 0$

   (b) f defined by $f(t) = 1$,

   (c) f defined by $f(t) = \sqrt{t}$.

298. A triangle has perimeter 12 units. Determine all possible values for the length of the shortest side, giving your answer as a set.

299. True or false?

   (a) Kites never have rotational symmetry,

   (b) All parallelograms have rotational symmetry,

   (c) A trapezium may have rotational symmetry.

300. Sketch the four graphs $y = \pm x^2$, $x = \pm y^2$ on the same axes, labelling the point of intersection in the positive quadrant.

——— End of 3rd Hundred ———